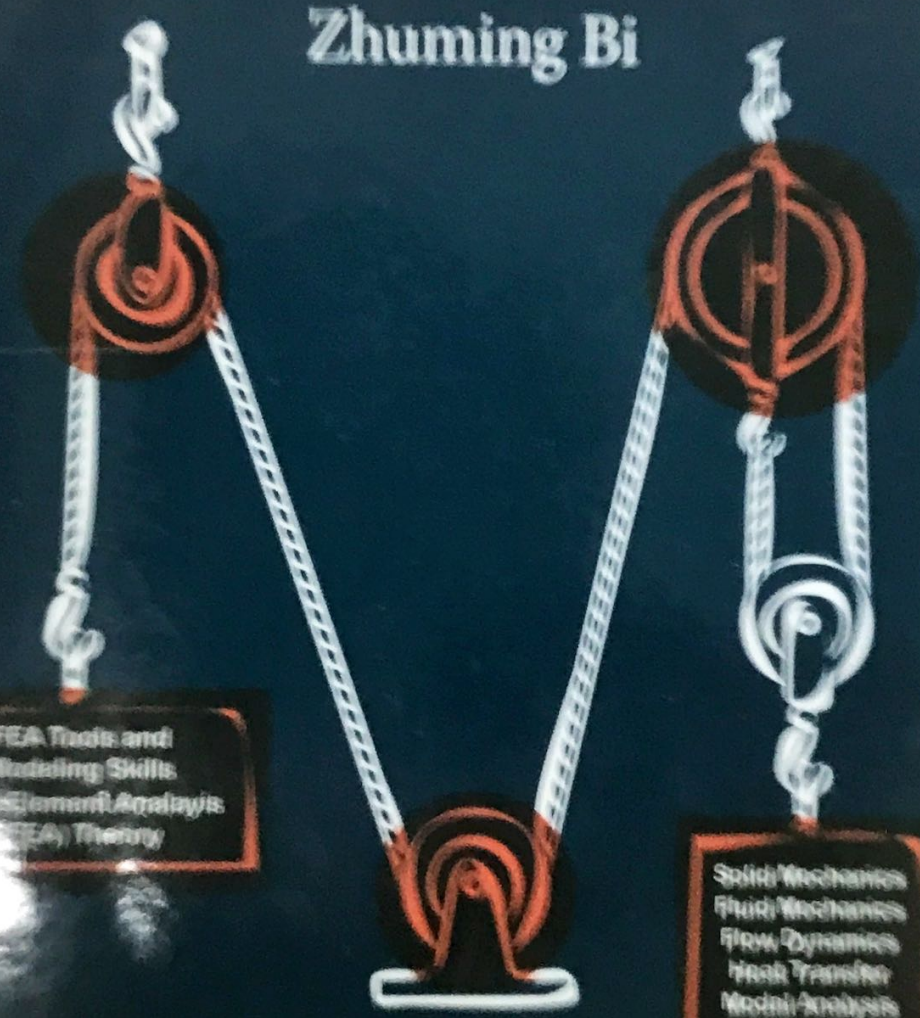


# Finite Element Analysis Applications

A SYSTEMATIC AND PRACTICAL APPROACH

Zhuming Bi



FEA Tools and  
Modeling Skills  
Thermal Analysis  
FEA Theory

Solid Mechanics  
Fluid Mechanics  
Flow Dynamics  
Heat Transfer  
Modal Analysis  
Multi-Physics  
Fatigue Analysis  
Electromagnetics  
Acoustics

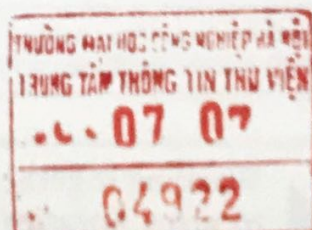


# Finite Element Analysis Applications

A Systematic and Practical Approach

**Zhuming Bi**

Purdue University Fort Wayne,  
Indiana, United States



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# Finite Element Analysis Applications

## A SYSTEMATIC AND PRACTICAL APPROACH

Application-oriented text that balances FEA theory with practical essentials

### Key features:

- Provides a systematic approach to dealing with the complexity of various engineering designs
- Includes sections on the design of machine elements to illustrate FEA applications
- Contains practical case studies presented as tutorials to facilitate the learning of FEA methods
- Ancillary materials include solutions manual for instructors, PPT lecture slides, and downloadable CAD models, for example in SOLIDWORKS®

Zhuming Bi's *Finite Element Analysis Applications: A Systematic and Practical Approach* strikes a solid balance between more traditional finite element analysis (FEA) textbooks that focus primarily on theory and the software-specific guidebooks that help teach students and professionals how to use particular FEA software packages without providing any of the theoretical foundation. In this new textbook, Professor Bi condenses the introduction of theories and focuses mainly on the essentials that students need to understand about FEA models. The book is organized to be application oriented, covering FEA modeling theory and skills directly associated with activities involved in design processes; discussion of such classic FEA elements such as truss, beam, and frame is limited. Through the use of several case studies, the book provides easy-to-follow guidance on modeling of different design problems. It uses SOLIDWORKS® simulation as the platform so that students do not need to waste time creating geometries for FEA modeling.

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